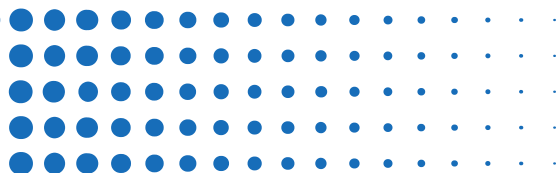


# ISC

**YEAR 2021**

## INDIAN SCHOOL CERTIFICATE EXAMINATION



## MATHEMATICS

**(860)**

February 2025

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## Council for the Indian School Certificate Examinations (CISCE)

### MISSION STATEMENT

*The Council for the Indian School Certificate Examinations is committed to serving the nation's children, through high quality educational endeavours, empowering them to contribute towards a humane, just and pluralistic society, promoting introspective living, by creating exciting learning opportunities, with a commitment to excellence.*

### ETHOS OF CISCE

- Trust and fair play.
- Minimum monitoring.
- Allowing schools to evolve their own niche.
- Catering to the needs of the children.
- Giving freedom to experiment with new ideas and practices.
- Diversity and plurality - the basic strength for evolution of ideas.
- Schools to motivate pupils towards the cultivation of:  
**Excellence** - The Indian and Global experience.  
**Values** - Spiritual and cultural - to be the bedrock of the educational experience.
- Schools to have an 'Indian Ethos', strong roots in the national psyche and be sensitive to national aspirations.

## MATHEMATICS (860)

**This subject may not be taken with Applied Mathematics.**

*(Note: For candidates who wish to pursue a career in Mathematics/ Physics/ Chemistry/ Engineering/ Architecture/ and other related fields.)*

### Aims

1. To enable candidates to acquire knowledge and to develop an understanding of the terms, concepts, symbols, definitions, principles, processes and formulae of Mathematics at the Senior Secondary stage.
2. To develop the ability to apply the knowledge and understanding of Mathematics to unfamiliar situations or to new problems.
3. To enhance ability of analytical and rational thinking in young minds.
4. To develop mathematical thinking and ability to communicate mathematical ideas logically and precisely.
5. To develop skills of –
  - a. Computation.
  - b. Logical thinking.
  - c. Handling abstractions.
  - d. Generalizing patterns.
  - e. Mathematical modeling to solve real-time problems.
  - f. Analyzing data and solving problems using multiple mathematical methods.
  - g. Reading and interpreting tables, charts, graphs, etc.
6. To enhance the ability to apply mathematical skills in interdisciplinary subjects
7. To develop an appreciation of the role of Mathematics in day-to-day life.
8. To develop a scientific attitude through the study of Mathematics.

## CLASS XI

There will be **two** papers in the subject:

**Paper I :** Theory (3 hours) .....80 marks

**Paper II:** Project Work .....20 marks

### PAPER I- THEORY: 80 Marks

#### DISTRIBUTION OF MARKS FOR THE THEORY PAPER

S.No.	UNIT	TOTAL WEIGHTAGE
1.	Sets and Functions	18 Marks
2.	Algebra	26 Marks
3.	Coordinate Geometry	20 Marks
4.	Calculus	8 Marks
5.	Statistics & Probability	8 Marks
	<b>TOTAL</b>	<b>80 Marks</b>

## 1. Sets and Functions

### (i) Sets

Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of a set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement of Sets.

### (ii) Relations & Functions

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto  $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$ ). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Function as a type of mapping, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions. Sum, difference, product and quotient of functions.

- **Sets:** Self-explanatory.
- **Basic concepts of Relations and Functions**
  - Ordered pairs, sets of ordered pairs.
  - Cartesian Product (Cross) of two sets, cardinal number of a cross product.
  - Relations as:
    - an association between two sets.
    - a subset of a Cross Product.
    - Domain, Range and Co-domain of a Relation.
  - Functions:
    - As special relations, concept of writing "y is a function of x" as  $y = f(x)$ .
    - Domain and range of a function
    - Reading, sketching and understanding the graphs of all standard real valued functions.

### (iii) Trigonometry

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity  $\sin^2 x + \cos^2 x = 1$ , for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing  $\sin(x \pm y)$  and  $\cos(x \pm y)$  in terms of  $\sin x$ ,  $\sin y$ ,  $\cos x$  &  $\cos y$  and their simple applications. Deducing the identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y},$$

$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

Identities related to  $\sin 2x$ ,  $\cos 2x$ ,  $\tan 2x$ ,  $\sin 3x$ ,  $\cos 3x$  and  $\tan 3x$ .

- **Angles and Arc lengths**
  - Angles: Convention of sign of angles.
  - Magnitude of an angle: Measures of Angles; Circular measure.
  - The relation  $S = r\theta$  where  $\theta$  is in radians. Relation between radians and degree.
  - Definition of trigonometric functions with the help of unit circle.
  - Truth of the identity  $\sin^2 x + \cos^2 x = 1$

**NOTE:** Questions on the area of a sector of a circle are required to be covered.

- **Trigonometric Functions**
  - Relationship between trigonometric functions.
  - Proving simple identities.
  - Signs of trigonometric functions.
  - Domain and range of the trigonometric functions.

- Trigonometric functions of all angles.
- Periods of trigonometric functions.
- Graphs of simple trigonometric functions (only sketches).

**NOTE:** Graphs of  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $\sec x$ ,  $\operatorname{cosec} x$  and  $\cot x$  are to be included.

- **Compound and multiple angles**

- Addition and subtraction formula:  $\sin(A \pm B)$ ;  $\cos(A \pm B)$ ;  $\tan(A \pm B)$ ;  $\tan(A + B + C)$  etc., Double angle, triple angle, half angle and one third angle formula as special cases.
- Sum and differences as products  
 $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$ , etc.
- Product to sum or difference  
i.e.  $2\sin A \cos B = \sin(A+B) + \sin(A-B)$  etc.

## 2. Algebra

### (i) Complex Numbers

Introduction of complex numbers and their representation, Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Square root of a complex number. Cube root of unity.

- Conjugate, modulus and argument of complex numbers and their properties.
- Sum, difference, product and quotient of two complex numbers additive and multiplicative inverse of a complex number.
- Square root of a complex number.
- Cube roots of unity and their properties.

### (ii) Quadratic Equations

Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients).

- Use of the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In solving quadratic equations.

- Equations reducible to quadratic form.
- Nature of roots

- Product and sum of roots.
- Roots are rational, irrational, equal, reciprocal, one square of the other.
- Complex roots.
- Framing quadratic equations with given roots.

**NOTE:** Questions on equations having common roots are to be covered.

- **Quadratic Functions.**

Given  $\alpha$ ,  $\beta$  as roots then find the equation whose roots are of the form  $\alpha^3, \beta^3$ , etc.

Case I:  $a > 0$   $\left\{ \begin{array}{l} \rightarrow \text{Real roots} \\ \rightarrow \text{Complex roots} \\ \rightarrow \text{Equal roots} \end{array} \right.$

Case II:  $a < 0$   $\left\{ \begin{array}{l} \rightarrow \text{Real roots} \\ \rightarrow \text{Complex roots,} \\ \rightarrow \text{Equal roots} \end{array} \right.$

Where 'a' is the coefficient of  $x^2$  in the equations of the form  $ax^2 + bx + c = 0$ .

- **Sign of quadratic**

Sign when the roots are real and when they are complex.

- Graph of quadratic function. Maximum/minimum value of quadratic function and value of  $x$  for which maximum/minimum occurs.

- **Inequalities**

- Linear Inequalities

Algebraic solutions of linear inequalities in one variable and their representation on the number line.

Self-explanatory.

- Quadratic Inequalities

Using method of intervals for solving problems of the type:

$$\begin{array}{c} x^2 + x - 6 \geq 0 \\ \begin{array}{c} + \quad \quad \quad - \quad \quad \quad + \\ \begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ | & & | \\ -3 & & 2 \end{array} \end{array} \end{array}$$

A perfect square e.g.  $x^2 - 6x + 9 \geq 0$ .

- Inequalities involving rational expression of type

$$\frac{f(x)}{g(x)} \leq a \text{ etc. to be covered.}$$

### (iii) Permutations and Combinations

Fundamental principle of counting. Factorial  $n$ . ( $n!$ ) Permutations and combinations, derivation of formulae for  ${}^n P_r$  and  ${}^n C_r$  and their connections, application.

- Factorial notation  $n!$ ,  $n! = n(n-1)!$
- Fundamental principle of counting.
- Permutations
  - ${}^n P_r$ .
  - Restricted permutation.
  - Certain things always occur together.
  - Certain things never occur.
  - Formation of numbers with digits.
  - Word building - repeated letters - No letters repeated.
  - Permutation of alike things.
  - Permutation of Repeated things.
  - Circular permutation – clockwise counterclockwise – Distinguishable / not distinguishable.
- Combinations
  - ${}^n C_r$ ,  ${}^n C_n = 1$ ,  ${}^n C_0 = 1$ ,  ${}^n C_r = {}^n C_{n-r}$ ,  
 ${}^n C_x = {}^n C_y$ , then  $x + y = n$  or  $x = y$ ,  
 ${}^{n+1} C_r = {}^n C_{r-1} + {}^n C_r$ .
  - When all things are different.
  - When all things are not different.
  - Mixed problems on permutation and combinations.

### (iv) Binomial Theorem

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term(s) in binomial expansion, applications.

- Significance of Pascal's triangle.
- Binomial theorem for positive integral powers,

$$\text{i.e. } (x + y)^n = {}^n C_0 x^n + {}^n C_1 x^{n-1} y + \dots + {}^n C_n y^n.$$

- Binomial coefficients.

Questions based on the above.

### (v) Sequence and Series

Sequence and Series. Arithmetic Progression (A.P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of first  $n$  terms of a G.P., infinite G.P. and its sum,

geometric mean (G.M.), relation between A.M. and G.M. Formulae for the following special sums  $\sum n$ ,  $\sum n^2$ ,  $\sum n^3$ .

- Arithmetic Progression (A.P.)
  - $T_n = a + (n - 1)d$
  - $S_n = \frac{n}{2} \{2a + (n - 1)d\}$
  - Arithmetic mean:  $2b = a + c$
  - Inserting two or more arithmetic means between any two numbers.
  - Three terms in A.P. :  $a - d$ ,  $a$ ,  $a + d$
  - Four terms in A.P.:  $a - 3d$ ,  $a - d$ ,  $a + d$ ,  $a + 3d$
- Geometric Progression (G.P.)
  - $T_n = ar^{n-1}$ ,
  - $S_n = \frac{a(r^n - 1)}{r - 1}$ ,  $|r| > 1$ ,  
 $S_n = \frac{a(1 - r^n)}{1 - r}$ ,  $|r| < 1$
  - $S_\infty = \frac{a}{1 - r}$ ;  $|r| < 1$
  - Geometric Mean,  $b = \sqrt{ac}$
  - Inserting two or more Geometric Means between any two numbers.
  - Three terms are in G.P.  $ar$ ,  $a$ ,  $ar^{-1}$
  - Four terms are in GP  $ar^3$ ,  $ar$ ,  $ar^{-1}$ ,  $ar^{-3}$
- Special sums  $\sum n$ ,  $\sum n^2$ ,  $\sum n^3$   
Using these summations to sum up other related expression.  
Finding  $n$ th. term of a sequence using Method of difference.

## 3. Coordinate Geometry

### (i) Straight Lines

Brief recall of two-dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point-slope form, slope-intercept form, two-point form, intercept form and normal form. General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.



- *Brief recall of basic concepts of Points and their coordinates.*
  - Section formula (internally/externally)
  - Coordinates of incentre, Area of triangle when vertices are given
  - Condition for collinearity of three points
- *The straight line*
  - Slope or gradient of a line.
  - Angle between two lines.
  - Condition of perpendicularity and parallelism.
  - Various forms of equation of lines.
  - Slope intercept form.
  - Two-point slope form.
  - Intercept form.
  - Perpendicular /normal form.
  - General equation of a line.
  - Distance of a point from a line.
  - Distance between parallel lines.
  - Equation of lines bisecting the angle between two lines.
  - Equation of family of lines
  - Definition of a locus.
  - Equation of a locus.

## (ii) Circles

- *Equations of a circle in:*
  - Standard form.
  - Diameter form.
  - General form.
  - Parametric form.
- *Given the equation of a circle, to find the centre and the radius.*
- *Finding the equation of a circle.*
  - Given three non collinear points.
  - Given other sufficient data for example centre is  $(h, k)$  and it lies on a line and two points on the circle are given, etc.
  - When circles touching each other externally/internally.
- *Intercepts made by the circle on the axes.*
- *Relative position of two circles.*

## (iii) Conic Section

Sections of a cone, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case

of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola.

- *Conics as a section of a cone.*
  - Definition of Foci, Directrix, Latus Rectum.
  - $PS = ePL$  where  $P$  is a point on the conics,  $S$  is the focus,  $PL$  is the perpendicular distance of the point from the directrix.
- (i) *Parabola*
  - $e = 1$ ,  $y^2 = \pm 4ax$ ,  $x^2 = 4ay$ ,  $y^2 = -4ax$ ,  $x^2 = -4ay$ .
  - Rough sketch of the above.
  - The latus rectum; quadrants they lie in; coordinates of focus and vertex; and equations of directrix and the axis.
  - Finding equation of Parabola when Foci and directrix are given, etc.
  - Application questions based on the above.

## (ii) Ellipse

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $e < 1$ ,  $b^2 = a^2(1 - e^2)$
- Cases when  $a > b$  and  $a < b$ .
- Rough sketch of the above.
- Major axis, minor axis; latus rectum; coordinates of vertices, focus and centre; and equations of directrices and the axes.
- Finding equation of ellipse when focus and directrix are given.
- Simple and direct questions based on the above.
- Focal property i.e.  $SP + SP' = 2a$ .

## (iii) Hyperbola

- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ,  $e > 1$ ,  $b^2 = a^2(e^2 - 1)$
- Cases when coefficient  $y^2$  is negative and coefficient of  $x^2$  is negative.



- Rough sketch of the above.
- Focal property i.e.  $SP - S'P = 2a$ .
- Transverse and Conjugate axes; Latus rectum; coordinates of vertices, foci and centre; and equations of the directrices and the axes.

#### (iv) Introduction to three-dimensional Geometry

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

- As an extension of 2-D
- Distance formula.
- Section and midpoint form

### 4. Calculus

#### (i) Limits and Derivatives

Derivative introduced as rate of change both as that of distance function and geometrically.

Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Definition of derivative relate it to slope of tangent of the curve, Derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

- Limits
  - Notion and meaning of limits.
  - Fundamental theorems on limits (statement only).
  - Existence of  $\lim_{x \rightarrow a} f(x)$
  - Left hand limit, Right hand limit
  - Limits of algebraic, trigonometric exponential and logarithmic functions.

**NOTE:** Indeterminate forms are to be introduced while calculating limits.

- Differentiation
  - Meaning and geometrical interpretation of derivative.
  - Derivatives of simple algebraic and trigonometric functions and their formulae.
  - Differentiation using first principles.
  - Derivatives of sum/difference.

- Derivatives of product of functions. Derivatives of quotients of functions.

### 5. Statistics and Probability

#### (i) Statistics

Measures of dispersion: range, mean deviation, variance and standard deviation of ungrouped/grouped data.

- Mean deviation about mean.
- Standard deviation - by direct method, short cut method and step deviation method.
- Combined mean and standard deviation

#### (ii) Probability

Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories studied in earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events.

- Random experiments and their outcomes.
- Events: sure events, impossible events, mutually exclusive and exhaustive events.
  - Definition of probability of an event
  - Laws of probability addition theorem.

## PAPER II

### PROJECT WORK – 20 Marks

Candidates will be expected to have completed two projects.

Mark allocation for **each** Project [10 marks]:

Overall format	1 mark
Content	4 marks
Findings	2 marks
Viva-voce based on the Project	3 marks
<b>Total</b>	<b>10 marks</b>

#### List of suggested assignments for Project Work:

1. Explore different methods to prove the result "If a set has 'n' number of elements, then the total number of subsets is  $2^n$ ".

2. Verify that for two sets A and B,  $n(A \times B) = pq$ , where  $n(A) = p$  and  $n(B) = q$ , the total number of relations from A to B is  $2^{pq}$ .
3. Using Venn diagram, verify the distributive law for three given non-empty sets A, B and C.
4. Identify distinction between a relation and a function with suitable examples and illustrate graphically.
5. Establish the relationship between the measure of an angle in degrees and in radians with suitable examples by drawing a rough sketch.
6. Illustrate with the help of a model, the values of sine and cosine functions for different angles which are multiples of  $\pi/2$  and  $\pi$ .
7. Draw the graphs of  $\sin x$ ,  $\sin 2x$ ,  $2 \sin x$ , and  $\sin x/2$  on the same graph using same coordinate axes and interpret the same.
8. Draw the graph of  $\cos x$ ,  $\cos 2x$ ,  $2 \cos x$ , and  $\cos x/2$  on the same graph using same coordinate axes and interpret the same.
9. Using argand plane, interpret geometrically, the meaning of  $i = \sqrt{-1}$  and its integral powers.
10. Draw the graph of quadratic function  $f(x) = ax^2 + bx + c$ . From the graph find maximum/minimum value of the function. Also determine the sign of the expression.
11. Construct a Pascal's triangle to write a binomial expansion for a given positive integral exponent.
12. Obtain a formula for the sum of the squares/sum of cubes of 'n' natural numbers.
13. Obtain the equation of the straight line in the normal form, for  $\alpha$  (the angle between the perpendicular to the line from the origin and the x-axis) for each of the following, on the same graph:
  - (i)  $\alpha < 90^\circ$
  - (ii)  $90^\circ < \alpha < 180^\circ$
  - (iii)  $180^\circ < \alpha < 270^\circ$
  - (iv)  $270^\circ < \alpha < 360^\circ$
14. Identify the variability and consistency of two sets of statistical data using the concept of coefficient of variation.
15. Construct the tree structure of the outcomes of a random experiment, when elementary events are not equally likely. Also construct a sample space by taking a suitable example.
16. Let S and S1 be two(non-concentric) circles with centres A , B and radii  $r_1$ ,  $r_2$  and  $d$  be the distance between their centres. Relation between  $r_1$ ,  $r_2$  and  $d$  with respect to relative position of two circles.
17. Construct different types of conics by PowerPoint Presentation, or by making a model, using the concept of double cone and a plane.
18. Use focal property of ellipse to construct ellipse.
19. Use focal property of hyperbola to construct hyperbola.
20. Write geometrical significance of X coordinate, Y coordinate, and Z coordinate in space. Using the above, find the distance of the point in space from x-axis/y-axis/z-axis. Explain the above using a three-dimensional model/ power point presentation.

## CLASS XII

There will be **two** papers in the subject:

**Paper I :** Theory (3 hours) .....80 marks

**Paper II:** Project Work .....20 marks

### PAPER I - THEORY: 80 Marks

#### DISTRIBUTION OF MARKS FOR THE THEORY PAPER

S.No.	UNIT	TOTAL WEIGHTAGE
1.	Relations and Functions	10 Marks
2.	Algebra	10 Marks
3.	Calculus	35 Marks
4	Vector Algebra	5 Marks
5	Three - Dimensional Geometry	6 Marks
6.	Linear Programming	5 Marks
7.	Probability	9 Marks
TOTAL		80 Marks

## 1. Relations and Functions

- (i) Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite function and inverse of a function.

- *Relations as:*

- *Relation on a set A*
- *Identity relation, empty relation, universal relation.*
- *Types of Relations: reflexive, symmetric, transitive and equivalence relation.*

- *Functions:*

- *As special relations, concept of writing "y is a function of x" as  $y = f(x)$ .*
- *Types: one to one, many to one, into, onto.*
- *Real Valued function.*
- *Domain and range of a function.*
- *Conditions of inevitability.*
- *Sketching of graph of a function and its inverse.*
- *Composite functions and Invertible functions (algebraic functions only).*

(ii) Inverse Trigonometric Functions

Definition, domain, range, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

- *Principal values.*
- *$\sin^{-1}x, \cos^{-1}x, \tan^{-1}x$  etc*
- *$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$ .*
- *$\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$ ;  $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$  and similar relations for  $\cot^{-1}x, \tan^{-1}x$ , etc.*

$$\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left(xy \mp \sqrt{1-y^2}\sqrt{1-x^2}\right)$$

$$\text{similarly } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}, xy < 1$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}, xy > -1$$

- *Formulae for  $2\sin^{-1}x, 2\cos^{-1}x, 2\tan^{-1}x, 3\tan^{-1}x$  etc. and application of these formulae.*

## 2. Algebra

### Matrices and Determinants

(i) Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order upto 3). Invertible matrices and proof of the uniqueness of inverse, if it exists (here all matrices will have real entries).

(ii) Determinants

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

- *Types of matrices ( $m \times n$ ;  $m, n \leq 3$ ), order; Diagonal matrix, Scalar matrix, Identity matrix, Triangular matrix.*
- *Symmetric, Skew symmetric matrices. Properties of Symmetric, Skew symmetric matrices.*
- *Operation – addition, subtraction, multiplication of a matrix with scalar, multiplication of two matrices (the compatibility).*

$$\text{E.g. } \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = AB(\text{say}) \text{ but } BA \text{ is not possible.}$$

- *Singular and non-singular matrices.*
- *Existence of two non-zero matrices whose product is a zero matrix.*

- Properties of adjoint of a square matrix.

- Inverse ( $2 \times 2$ ,  $3 \times 3$ )  $A^{-1} = \frac{\text{Adj}A}{|A|}$

- Properties of inverse

- **Martin's Rule (i.e. using matrices)**

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

Problems based on above.

**NOTE:** The conditions for consistency of equations in two and three variables, using matrices, are to be covered.

- **Determinants**

- Order.

- Minors.

- Cofactors.

- Expansion.

- Applications of determinants in finding the area of triangle and collinearity.

- Properties of determinants. Problems based on properties of determinants.

### 3. Calculus

- (i) **Continuity, Differentiability and Differentiation.** Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

- **Continuity**

- Continuity of a function at a point  $x = a$ .

- Continuity of a function in an interval.

- Algebra of continuous function.

- Removable discontinuity. Types of removable discontinuity.

- **Differentiation**

- Concept of continuity and differentiability of  $|x|$ ,  $[x]$ , etc.

- Derivatives of trigonometric functions.

- Derivatives of exponential functions.

- Derivatives of logarithmic functions.

- Derivatives of inverse trigonometric functions - differentiation by means of substitution.

- Derivatives of implicit functions and chain rule.

- Derivatives of Parametric functions.

- Differentiation of a function with respect to another function e.g. differentiation of  $\sin x^3$  with respect to  $x^3$ .

- Logarithmic Differentiation - Finding  $dy/dx$  when  $y = x^{x^{x^{\dots}}}$ .

- Successive differentiation up to 2<sup>nd</sup> order.

**NOTE:** Derivatives of composite functions using chain rule.

#### (ii) Applications of Derivatives

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangent and normal, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Problems that illustrate basic principles and understanding of the subject as well as real-life situations.

- **Equation of Tangent and Normal, Angle between two curves**

- **Rate measure.**

- **Increasing and decreasing functions.**

- **Maxima and minima.**

- Critical points, Stationary/turning points, Extreme points.

- Absolute maxima/minima

- local maxima/minima
- First derivatives test and second derivatives test
- Application problems based on maxima and minima.

### (iii) Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

#### • Indefinite integral

- Integration as the inverse of differentiation. (anti-derivative).
- Anti-derivatives of polynomials and functions  $(ax + b)^n$ ,  $\sin x$ ,  $\cos x$ ,  $\sec^2 x$ ,  $\csc^2 x$  etc.
- Integrals of the type  $\sin^2 x$ ,  $\sin^3 x$ ,  $\sin^4 x$ ,  $\cos^2 x$ ,  $\cos^3 x$ ,  $\cos^4 x$ .
- Integration of  $1/x$ ,  $e^x$ .
- Integration by substitution.
- Integrals of the type  $f'(x)[f(x)]^n$ ,  $\frac{f'(x)}{f(x)}$ .
- Integration of  $\tan x$ ,  $\cot x$ ,  $\sec x$ ,  $\csc x$ .
- Integration by parts.
- Integration using partial fractions.

Expressions of the form  $\frac{f(x)}{g(x)}$   
when degree of  $f(x) <$  degree of  $g(x)$

$$\begin{aligned} \text{E.g. } \frac{x+2}{(x-3)(x+1)} &= \frac{A}{x-3} + \frac{B}{x+1} \\ \frac{x+2}{(x-2)(x-1)^2} &= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} \\ \frac{x+1}{(x^2+3)(x-1)} &= \frac{Ax+B}{x^2+3} + \frac{C}{x-1} \end{aligned}$$

When degree of  $f(x) \geq$  degree of  $g(x)$ ,

$$\text{e.g. } \frac{x^2+1}{x^2+3x+2} = 1 - \left( \frac{3x+1}{x^2+3x+2} \right)$$

- Integrals of the type:

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$$

and  $\int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx,$

$\int \sqrt{ax^2+bx+c} dx, \int (px+q)\sqrt{ax^2+bx+c} dx,$   
integrations reducible to the above forms.

$$\begin{aligned} &\int \frac{dx}{a \cos x + b \sin x}, \\ &\int \frac{dx}{a+b \cos x}, \int \frac{dx}{a+b \sin x}, \int \frac{dx}{a \cos x + b \sin x + c}, \\ &\int \frac{(a \cos x + b \sin x) dx}{c \cos x + d \sin x}, \\ &\int \frac{dx}{a \cos^2 x + b \sin^2 x + c} \\ &\int \frac{1 \pm x^2}{1+x^4} dx, \\ &\int \frac{dx}{1+x^4}, \int \sqrt{\tan x} dx, \int \sqrt{\cot x} dx \text{ etc.} \end{aligned}$$

#### • Definite Integral

- Fundamental theorem of calculus (without proof)
- Properties of definite integrals.
- Problems based on the following properties of definite integrals are to be covered.

$$\int_a^b f(x) dx = \int_a^b f(t) dt$$

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

where  $a < c < b$

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$2a \int_0^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0, & f(2a-x) = -f(x) \end{cases}$$



$$\int_{-a}^a f(x)dx = \begin{cases} 2 \int_0^a f(x)dx, & \text{if } f \text{ is an even function} \\ 0, & \text{if } f \text{ is an odd function} \end{cases}$$

(iv) Application of Integrals (Area under the curve) Application in finding the area bounded by simple curves and coordinate axes. Area enclosed between two curves.

- Application of definite integrals - area bounded by curves, lines and coordinate axes is required to be covered.
- Simple curves: lines, circles/ parabolas/ ellipses, polynomial functions, modulus function, exponential function, logarithmic function.

(v) Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:  $\frac{dy}{dx} + py = q$ , where p and q are

functions of x or constants.  $\frac{dx}{dy} + px = q$ ,

where p and q are functions of y or constants.

- Differential equations, order and degree.
- Formation of differential equation by eliminating arbitrary constant(s).
- Solution of differential equations.
- Variable separable.
- Homogeneous equations.
- Linear form  $\frac{dy}{dx} + Py = Q$  where P and Q are functions of x/constant. Similarly, for  $\frac{dx}{dy}$ .

**NOTE 1:** Equations reducible to variable separable type are included.

**NOTE 2:** The second order differential equations are excluded.

#### 4. Vector Algebra

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector,

components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

- As directed line segments.
- Magnitude and direction of a vector.
- Types: equal vectors, unit vectors, zero vector.
- Position vector.
- Components of a vector.
- Vectors in two and three dimensions.
- $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$  as unit vectors along the x, y and the z axes; expressing a vector in terms of the unit vectors.
- Operations: Sum and Difference of vectors; scalar multiplication of a vector.
- Section formula.
- Scalar (dot) product of vectors and its geometrical significance.
- Scalar and vector projection.
- Cross product and its geometrical significance. Its properties - area of a triangle, area of parallelogram, collinear vectors.

**NOTE: Proofs of geometrical theorems by using Vector algebra are excluded.**

#### 5. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

- Equation of x-axis, y-axis, z axis and lines parallel to them.
- Equation of xy - plane, yz - plane, zx - plane.
- Direction cosines, direction ratios.
- Angle between two lines in terms of direction cosines /direction ratios.
- Condition for lines to be perpendicular/ parallel.
- Lines
  - Cartesian and vector equations of a line through one and two points.



- Coplanar and skew lines.
- Conditions for intersection of two lines.
- Distance of a point from a line.
- Shortest distance between two lines.

• Planes

- Cartesian and vector equation of a plane.
- Direction ratios of the normal to the plane.
- One point form.
- Normal form.
- Intercept form.
- Distance of a point from a plane.
- Intersection of the line and plane.
- Angle between two planes, a line and a plane.

## 6. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded and unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

*Introduction, definition of related terminology such as constraints, objective function, optimization, advantages of linear programming; limitations of linear programming; application areas of linear programming; different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible (bounded/ unbounded) and infeasible regions, feasible and infeasible solutions, optimum feasible solution(may/may not exists).*

## 7. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

- Independent and dependent events conditional events.
- Laws of Probability, addition theorem, multiplication theorem, conditional probability.
- Theorem of Total Probability.

- Baye's theorem.
- Theoretical probability distribution, probability distribution function; mean of random variable.

## PAPER II PROJECT WORK – 20 Marks

Candidates will be expected to have completed two projects.

**The project work will be assessed by the subject teacher and a Visiting Examiner appointed locally and approved by the Council.**

Mark allocation for **each** Project [10 marks]:

Overall format	1 mark
Content	4 marks
Findings	2 marks
Viva-voce based on the Project	3 marks
<b>Total</b>	<b>10 marks</b>

### List of suggested assignments for Project Work:

1. Using a graph, demonstrate a function which is one-one but not onto.
2. Using a graph demonstrate a function which is invertible.
3. Draw the graph of  $y = \sin^{-1} x$  (or any other inverse trigonometric function), using the graph of  $y = \sin x$  (or any other relevant trigonometric function). Demonstrate the concept of mirror line (about  $y = x$ ) and find its domain and range.
4. Explore the principal value of the function  $\sin^{-1} x$  (or any other inverse trigonometric function) using a unit circle.
5. Find the derivatives of a determinant of the order of  $3 \times 3$  and verify the same by other methods.
6. Verify the consistency of the system of three linear equations of two variables and verify the same graphically. Give its geometrical interpretation.
7. For a dependent system (non-homogeneous) of three linear equations of three variables, identify infinite number of solutions.
8. Explain the concepts of increasing and decreasing functions, using geometrical significance of  $dy/dx$ . Illustrate with proper examples.

9. Explain the geometrical significance of point of inflexion with examples and illustrate it using graphs.
10. Explain and illustrate (with suitable examples) the concept of local maxima and local minima using graph.
11. Explain and illustrate (with suitable examples) the concept of absolute maxima and absolute minima using graph.
12. Explain the conditional probability, the theorem of total probability and the concept of Bayes' theorem with suitable examples.
13. Explain the types of probability distributions and derive mean and variance of binomial probability distribution for a given function.
14. Using any suitable data, find the minimum cost by applying the concept of Transportation problem.
15. Using any suitable data, find the minimum cost and maximum nutritional value by applying the concept of Diet problem.
16. Using any suitable data, find the Optimum cost in the manufacturing problem by formulating a linear programming problem (LPP).
17. Demonstrate application of differential equations to solve a given problem (example, population increase or decrease, bacteria count in a culture, etc.).
18. Using vector algebra, find the area of a parallelogram/triangle. Also, derive the area analytically and verify the same.
19. Using Vector algebra, prove the formulae of properties of triangles (sine/cosine rule, etc.)
20. Using Vector algebra, prove the formulae of compound angles, e.g.  $\sin(A + B) = \sin A \cos B + \sin B \cos A$ , etc.
21. Find the image of a line with respect to a given plane.
22. Find the distance of a point from a given plane measured parallel to a given line.
23. Find the distance of a point from a line measured parallel to a given plane.
24. Find the area bounded by a parabola and an oblique line.
25. Find the area bounded by a circle and an oblique line.
26. Find the area bounded by an ellipse and an oblique line.
27. Find the area bounded by a circle and a circle.
28. Find the area bounded by a parabola and a parabola.
29. Find the area bounded by a circle and a parabola.  
**(Any other pair of curves which are specified in the syllabus may also be taken.)**
30. Analyse - Three methods (proofs) to find the area under the curve.
31. Tessellation:  
Types of tessellations, Geometrical shape of tessellation, Tessellation in nature, Man made tessellation, application in real life situation.
32. Scared Geometry and Euclid geometry comparison and different approach (Mathematically)
33. Derivation of  $e^{\int p dx}$  and its applications.
34. Differentiation – instantaneous rate of change of displacement, why not rate of change of one variable with reference to other variable.
35. Derivation of determinant.

**NOTE:** No question paper for Project Work will be set by the CISCE.

### SAMPLE TABLE FOR PROJECT WORK

S. No.	Unique Identification Number (Unique ID) of the candidate	<u>PROJECT 1</u>					<u>PROJECT 2</u>					TOTAL MARKS
		A	B	C	D	E	F	G	H	I	J	(E + J)
		Teacher	Visiting Examiner	Average Marks (A + B ÷ 2)	Viva-Voce by Visiting Examiner	Total Marks (C + D)	Teacher	Visiting Examiner	Average Marks (F + G ÷ 2)	Viva-Voce by Visiting Examiner	Total Marks (H + I)	
		7 Marks*	7 Marks*	7 Marks	3 Marks	10 Marks	7 Marks*	7 Marks*	7 Marks	3 Marks	10 Marks	20 Marks
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												

*Breakup of 7 Marks to be awarded separately by the Teacher and the Visiting Examiner is as follows:		Name of Teacher: Signature:	Date
Overall Format	1 Mark		
Content	4 Marks	Name of Visiting Examiner  Signature:	Date
Findings	2 Marks		

NOTE: VIVA-VOCE (3 Marks) for each Project is to be conducted only by the Visiting Examiner, and should be based on the Project only